SOME QUESTIONS RELATING TO THE CONTROL OF FLOWS NEAR A WALL

V. G. Bogdevich, G. F. Kobets,
G. S. Kozyuk, G. S. Migirenko,
V. I. Mikuta, B. P. Mironov,
B. G. Novikov, V. A. Tétyanko,
and Yu. V. Shtatnov

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The present paper is a brief survey of investigations whose formulation and conduct were organized by Mikhail Alekseevich Lavrent'ev and most of which were carried out under his direct guidance. In these investigations a great deal of attention was given to cavitation flows in a heavy liquid, the properties of the transition from laminar to turbulent flow, the laminarization of flows near a wall on permeable surfaces, and the action of gas bubbles and polymer additives on the structure of turbulence near a wall.

Investigations of Cavitation Flows in a Heavy Liquid. A characteristic property of cavitation flows in a heavy liquid is the distortion of the shape of the free surface of a cavity even for very large values of the Froude number, a distortion caused not only by the buoyancy forces directly but also by a diminishing vortex sheet. This distortion of the cavity shape leads to so-called "flotation of the cavity," "loss of buoyancy," etc. In our investigations the surface of the cavitator and the surface of the cavity were regarded as a unified boundary surface of the current, on part of which the pressure is constant. This made it possible to reduce the problem of cavitation in a heavy liquid to the thoroughly studied problem of nonseparating flow past an object in a weightless liquid, to explain the reasons for the occurrence of such phenomena as "flotation of the cavity," and "loss of buoyancy," and to extend the Kutta–Joukowski formula to the case of cavitation flows in a heavy liquid [1]

$$Y = \gamma D - \rho U_\infty \iiint_\omega \Omega_z d\omega,$$

where Y is the vertical component of the resulting external forces acting on the boundary surface of the current; Ω_z, component of the curl of the velocity in the direction of the horizontal axis normal to the velocity of the flow; ω, volume of the liquid space included in the volume bounded by the current surface.

Moreover, we were able to predict a number of previously unknown effects and confirm experimentally the correctness of these predictions. In particular, we predicted, and confirmed experimentally, that a cone suspended in water at a zero angle of attack will sink as soon as a cavity is formed behind it and that there must be some "nonfloating" cavities in a heavy liquid. A cavity behind an inclined disk will also sink if the lifting force on the disk exceeds in weight the water displaced by the cavity. We predicted the existence in a heavy liquid of cavitation flows with mirror symmetry with respect to a vertical plane. In the plane case, theorems concerning the existence and uniqueness of solutions have been proved for such flows [2]. We predicted the existence of cavitation flows when there are no diminishing vortices at all behind the boundary surface of the current. These flows were called circulation-free flows. We confirmed experimentally for three-dimensional flows the hypothesis that the nature of the flow in the rear part has only a slight effect on the shape of the free surface of the cavity up to its maximum cross section [3-7]. For the solution of this problem, we developed a method for measuring the shape of the free surface of the cavity by means of amorphous soluble rods. Knowing the shape of the three-dimensional cavities, we were able to keep the gaps in the cavity very small and make sure that there was a "film" closure over the entire contour and to obtain values of the order of 10^{-4} for the flow-rate coefficients of the ventilated gas, $C_Q = Q/U_\infty S$, where Q is the volumetric flow rate of the gas, U_∞ is the velocity of the flow, and S is the area of the middle section of the boundary surface of the current. The length of the cavity was 0.6-0.8 times the chord length L of the boundary surface of the current, the Reynolds number $Re = U_\infty L/\nu = 4 \cdot 10^6$, the Froude number $Fr = U_\infty/\sqrt{gL} = 1.05$, and the free surface of the cavity was mirror-smooth everywhere.

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We studied experimentally the nature of the flow behind the point of closure of the cavity, when the cavity approaches the wall at a large angle and the flow at the point of closure of the cavity is nonstationary. We measured the pressure distributions, the pulsations in pressure, and other characteristics of two-phase flow behind the point of closure of the cavity [3].

We studied cavitation flows for the case of closure of a cavity onto a liquid jet, when without the jet the flow at the point of closure of the cavity is nonstationary in character. It was shown that the nonstationarity of the flow at the point of closure of the cavity leads to strong perturbations of the entire free surface. In this case the closure of the cavity onto a jet substantially stabilizes the flow at the point of closure, makes the free surface of the cavity less perturbed, reduces the flow rate of the air ventilated through the cavity, and precludes any separation in the flow behind the point of closure of the cavity.

We studied the effect of the nature of the flow in the wall layer of the cavitator on the perturbations of the free surface of the cavity [1]. It was shown that if there is stable laminar flow on the cavitator, then in the absence of other sources of perturbation, the entire free surface of the cavity will be mirror-smooth. For Reynolds number values somewhat higher than the critical value, i.e., for cases when the point of loss of stability of the laminar boundary layer is in front of the edge of the cavitator, there will be regular waves on the free surface of the cavity, and their length will correspond to the length of an unstable wave at the point of loss of stability. As the Reynolds number increases further, the regularity of the perturbations is violated, and the perturbations in the free surface of the cavity become chaotic. Placing a turbulizer on the cavitator causes irregular perturbations on the free surface, and the cavity becomes nontransparent. If there is unstable laminar flow at the edge of the cavitator, then pumping off a small part of the liquid makes the free surface of the cavity mirror-smooth. In the case of a film closure the flow rate of ventilated air is reduced by a factor of 3-4. For free cavities the perturbation of the cavity surface has no effect on the gas flow rate.

If the cavity is separated by a horizontal plate into two parts, we find that for small values of the Froude number the perturbations of the lower and upper free surfaces of the cavity differ sharply [3]. In the upper part there will be a predominance of long-wave regular perturbations, and in the lower part a predominance of short-wave irregular perturbations.

Investigations of fully developed natural cavitation showed that evaporative cooling of the boundaries of a cavity and changes in saturated-vapor pressure resulting from the curvature of the free surface cannot substantially alter the dimensions of the cavity [10]. If there are obstructions behind the cavitator, the cavity shape found will depend on the nature of the flow development.

In [2] Mal'tsev proved a constructive theorem for the existence and uniqueness of a solution (for large Froude numbers) of the plane cavitation problem of flow of a heavy liquid past an arc set on a rectilinear horizontal bottom, where the law of distribution of the flow velocity values is given. A number of plane problems concerning the closure of a cavity onto a wall jet of liquid were solved in [11]. Necessary and sufficient conditions for the unique solvability of the boundary-value problem of constructing an analytic function completely defined on part of the boundary were obtained in [12]. The applications considered were the problems of determining the shape of a confusor with a given velocity profile at the outlet and the problem of finding the shape of a cavitator from the known geometry of the cavity formed behind it.

Within the framework of the model of an ideal incompressible liquid, we worked out methods for solving in the exact formulation a number of plane and axisymmetric cavitation problems with a given law of distribution of the velocity on part of the surface.

The first method for solving plane problems with a partially unknown boundary is based on the ideas of the method of finite-dimensional approximation proposed in [13] in the investigation of questions of the existence and uniqueness of the solution of problems with free boundaries. In the proposed method the approximation of a given curvilinear arc is carried out by using a piecewise smooth curve with a continuous derivative at the nodal points. The parameters appearing in the analytic solution of the problem are determined from a system of transcendental equations obtained from the condition that the boundary conditions are satisfied at the nodal points. The first problem to which this method was applied was the problem of cavitation flow of a heavy liquid past a curvilinear projection situated on a horizontal bottom [14, 15]. By applying Lavrent'ev's comparison theorem [16], we obtained an estimate for the range of allowable Froude numbers corresponding to the given cavitation number. It was shown that for small Froude numbers the length of the cavity may increase to several times its length in the weightless case.

Subsequently, the method was used for solving problems with stoppage points on the desired boundary [17, 18], which had previously been considered only in an approximate formulation. The calculations of [19] confirmed the conclusions of the linear and approximate theories concerning the existence of two solutions of problems relating to cavitation flow past an obstacle in a longitudinal gravitational field.

The second method of solving problems with free surfaces is related to vortex-layer methods. The integral representation of the stream function of plane and axisymmetric flows is obtained as a result of the continuous distribution of the vortex layer along the entire boundary surface. The requirement that the boundary conditions of the problem must be satisfied leads to a system of functional equations, from which we determine the value of the velocity along given segments of the boundary and also determine the function describing the shape of the free boundary. The desired functions can be represented as spline functions of third order. The later stage of the solution involves the use of the collocation method. We obtain a system of transcendental equations which can be solved by using methods of the Newton type. The method has been used for solving axisymmetric problems of cavitation flow past solids of revolution [20, 21], for the analysis of the effect of tube walls on the parameters of axisymmetric cavities [22], and in the problem of the effect of a longitudinal gravitational field on plane and axisymmetric cavitation flows [23].

Bubble-Type Gas Saturation of Flows near a Wall. If gas is blown into a current of a dropwise liquid, it is possible — owing to the substantial change in the density and viscosity of the mixture, depending on the concentration of the bubbles, and also to the introduction of additional linear and time scales — to affect the characteristics of flows near a wall over a wide range of values. It is known that in this type of flow the hydraulic resistance and friction may be greater or smaller by a large factor than for the flow of a homogeneous liquid.

The variation of this effect as a function of the structure of a flow with bubble-type gas saturation has been investigated experimentally and numerically by using the example of a turbulent boundary layer. Extensive experimental investigations have been carried out on its characteristics both when there is a distributed input of gas through a permeable surface and when the input of gas is discrete, passing through narrow bands of perforations and cracks [24, 25]. It was found that such characteristics as the intensity of pressure pulsations and the friction at the wall depend on the concentration of bubbles in the wall zone of the boundary layer, where the velocity profile is described by a logarithmic law. The characteristic parameter is the maximum value of the volumetric concentration of bubbles, φ_{max}. Investigations have been conducted on the variation of this parameter as a function of the gas input rate and the velocity of the unperturbed flow. When a limiting value of $\varphi_{max} \approx 0.75$ is reached (corresponding to close packing of spheres), the current is forced almost completely away from the wall.

A formula was found for the variation of $\overline{c}_f'$ as a function of φ_{max}: $\overline{c}_f' = \overline{c}_{f(o)}'[1 - (\varphi_{max} - \varphi_o)]^2$ (Fig. 1). It was found that there exists a threshold value of concentration $\varphi_{max} \equiv \varphi_o \approx 0.1$ below which the gas saturation does not effect the coefficient of friction. When φ_{max} attains a certain value, for a given velocity of flow, there is an instability of the bubble layer, which is manifested in the minimum points of the function $\overline{c}_f'(\varphi_{max})$. This phenomenon is accompanied by a considerable increase in low-frequency perturbations. The conditions for its occurrence are determined by Kutateladze's hydrodynamic stability criterion [26]

$$K = \rho_2^{0.5} v_{2\,cr} / [g\sigma(\rho_1 - \rho_2)]^{0.25},$$

where ρ_1 and ρ_2 are the densities of the heavy and light phases, respectively; g, acceleration of gravity; σ, coefficient of surface tension; and v_{2cr}, critical value of the characteristic velocity of the light phase.

A comparison of the results of the numerical solution of the problem of nonuniform gas input into a turbulent boundary layer, within the framework of the single-velocity model of the flow of a quasihomogeneous medium, with the experimental data for a gradient-free boundary layer showed that an acceptable accuracy can be achieved for flow velocities of the main current exceeding 2 m/sec.

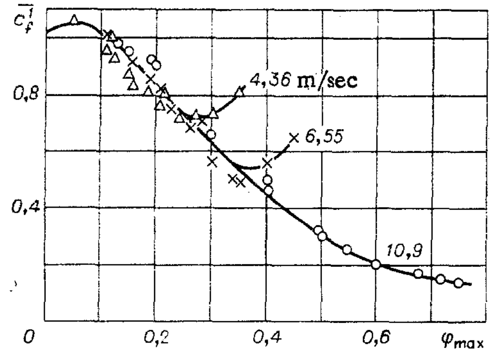

Fig. 1

Taking account of this limitation on the velocity of the flow, it should be noted that the Reynolds numbers calculated from the diameter of a bubble and from the distance from the wall of the level with the maximum bubble concentration are constant, which is characteristic for self-similarity conditions.

Transition from Laminar to Turbulent Flow. On the basis of the method of small oscillations and previously obtained experimental data [27-31], we proposed a qualitative scheme for the mechanism of occurrence and development of the transition from laminar to turbulent flow [32]. The principal role in this scheme is played by the critical layer, in which the velocity of the main flow coincides with the phase velocity of the perturbation. It is assumed that in the critical layer, as a result of the Tolmin-Schlichting law, there is a primary discrete vortex which interacts with the main flow and moves into layers with higher vorticity, i.e., to the wall. As a result, a complete system of vortices appears. This scheme made it possible to explain some known facts and also to predict the presence of characteristic regions along the length and thickness of the transtiion zone [33, 34].

Recent experimental material on probability density distributions for velocity pulsations in the transition zone has made it possible to determine quantitatively the relative positions of three characteristic regions along the thickness of the boundary layer; these regions had been introduced earlier on the basis of a qualitative analysis of the realizations of velocity pulsations [32]. In addition, an interesting law was established for the variation of the coefficient of ν_1, of the velocity pulsations along the length and thickness of the zone of transition from laminar to turbulent flow [Fig. 2, where: a) γ = 0, 0.08, 0.20, 0.45, 0.65, and 0.90 (points 1-6, respectively); b) Δf = 1 Hz to 10 kHz; 1 Hz to 1 kHz, and 1-10 kHz (points 1-3, respectively), with 4 representing coinciding points]. For a mobility coefficient γ_1 of the order of 0.1, we observe a sharp increase in the coefficient of asymmetry (Fig. 2a), which takes on large positive values that subsequently decrease as the transition develops. For $\gamma_1 > 0.8$ the coefficient of asymmetry becomes negative, as in the pretransition layer. It should be noted that at a distance of a displacement thickness we find that the coefficient ν_1 takes on a zero value. At this same distance, at each cross section, the mobility coefficient takes on its maximum value.

Figure 2b shows ν_1 as a function of the width of the frequency band of the investigated signal of velocity pulsations at the cross section with $\gamma_1 = 0.58$. It can be seen that the positive values of ν_1 for $y \leqslant \delta^*$ are determined by the low-frequency components of the velocity pulsations.

Many investigators have been looking into the problem of the downstream shift of the laminar-to-turbulent transition in order to obtain a laminarized boundary layer. One way to solve this problem is to use distributed pumping from the boundary layer. Viewpoints differ concerning the advisability of using a uniform distribution or a stepwise distribution of the pumping.

We conducted a special investigation of the effect of distributed pumping on the structure of a flow in the boundary layer [29, 35-37]. In one of the series of experiments we used a wedge-shaped model with a plane working surface made of a porous material; the model had eight sections with different regulation of the amount of air pumped out. We selected experimentally a stepwise law for the distribution of the pumping along the sections, leading to laminarization of the boundary layer on the entire working surface. In a second variant we used uniformly distributed pumping over all the sections, with the same total amount of air pumped out as in the first variant.

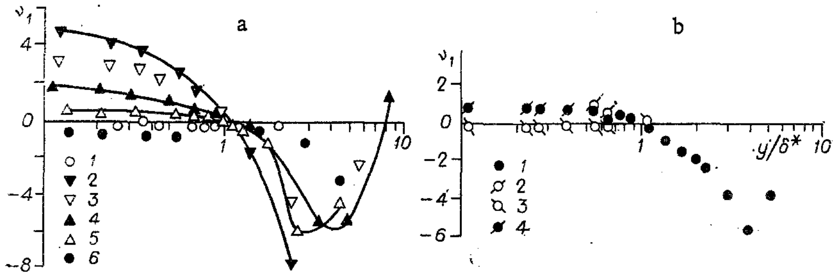

Fig. 2

In the first variant, in the region of the first two sections, the flow became strongly stabilized as a result of intensive pumping in the transition zone and the beginning of the turbulent boundary layer. Farther downstream the boundary layer develops further, and the intensity and spectral composition of the velocity pulsations are close to laminar up to the last working section. In the second variant there is some delay in the transition, and thereafter a turbulent boundary layer with new characteristics is formed. The velocity profiles along the length agreed within the limits of experimental accuracy, and the thickness of the layer itself remained constant. In this layer we observed a reduction in the intensity of velocity pulsations by a factor of 2-3 in comparison with the turbulent boundary layer with no pumping, although the relative spectral composition of the velocity pulsations remained practically the same. The increase in the amount of liquid in the sample within the limits of the available power values, for uniformly distributed pumping, did not lead to laminarized flow.

The experiments show that for laminarization of the flow in the boundary layer, it is desirable to use a specified law of stepwise distribution of the pumping along the length of the surface, in order to reduce the amount of liquid extracted. A uniform distributed pumping for moderate amounts of extracted liquid leads to a turbulent boundary layer with a lower level of velocity pulsations, which may be of interest for the study of hydrodynamic noise in a turbulent boundary layer. Our investigation shows that the law governing the distribution of the pumping along the length of the surface must be chosen in accordance with the formulation of the problem.

Laminarization of Wall Flows. We numerically and experimentally investigated the possibility of obtaining on contours of revolution a stable laminar asymptotic boundary layer, which, for a constant value of the parameter U_∞/ν and a given permeable material, makes it possible at least to preserve the relative length of the laminar flow for an arbitrarily large increase in the scale of the contour [38, 39]. As the permeable material, we use perforated shells with hole diameters of 0.06-0.08 mm and a spacing of 0.5 mm between holes. Under the permeable shell we placed a single section so profiled that the pumping was uniform on the entire permeable surface under the conditions of the experiment. The equation of a generator of the contour of revolution has the form $[(x - 0.42)/0.42]^2 + (r/0.07142856)^2 = 1$ for $0 \leqslant x \leqslant 0.63$, $r = 0.273401x^4 - 0.451668x^3 - 0.109136x^2 + 0.303670x - 0.0162679$ for $0.63 \leqslant x \leqslant 1$. The length of the perforated shell was 0.75 times the length of the contour past which the flow was taking place.

The investigations were carried out in the low-turbulence wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Academy of Sciences of the USSR, at a flow velocity of 50 to 105 m/sec and Reynolds number values of 10^7 along the length of the contour, and in water for Reynolds number values of 10^8. In the wind tunnel the wall flow was laminar even at pumping-coefficient values of $C_Q = v_0/U_\infty = 2\cdot10^{-4}$, at least over the entire length of the permeable shell, and we found a maximum reduction in the total resistance. The intensity of the velocity pulsations on some perforated sections decreased sharply with increasing ordinate x, while on others it increased. Such behavior in the intensity of the velocity pulsations can be attributed to the fact that the roughness varied from one perforated section to the next, and the laminar wall flow was stable.

For $C_Q = 5.3\cdot10^{-4}$ on the segment $0.6 \leqslant x \leqslant 0.75$ the velocity profile was asymptotic and the value of Re* changed very little. We also conducted investigations of the effect produced on the length of the laminar flow by such turbulizing factors as external turbulence, the shape of the input segment of the perforation holes, the roughness of the perforated shell, the indentations and projections at the joints between adjacent sections, the degree of obstruction in the shells, etc. [39].

TABLE 1

$C_Q\cdot 10^4$	U_∞, m/sec	$C_{x\,tot}\cdot 10^4$	$C_{x\,eff1}\cdot 10^4$	$C_{x\,eff2}\cdot 10^4$
0	13,5	30,0	30,0	30,0
3,5	15,3	12,1	8,6	10,9
3,4	15,5	20,9	17,6	19,8
3,7	15,8	11,1	7,4	10,1

The results of the experimental investigations for Re = 10^8, shown in Table 1, confirmed that the distributed pumping has a very strong effect on the characteristics of the wall flow. In the table we used the following notation: $C_{xtot} = 2X_{tot}/\rho U_\infty^2 S$, coefficient of total resistance; $C_{xeff} = C_{xtot} - C_Q$, effective coefficient of resistance; $C_{xeff_2} = C_{xeff} - C\Delta p$, effective coefficient of resistance when account is taken of the hydraulic losses in the pumping system; S, area of the wetted surface.

We investigated the characteristics of turbulent flow between two coaxial perforated cylinders when there was a positive radial component of velocity v_r [39]. The inner cylinder, with a diameter of 250 mm, was fixed, while the outer cylinder, 500 mm in diameter, rotated with a linear velocity U_0 ranging from 7 to 22 m/sec. Without pumping, for $v_r = 0$, the profile of the tangential component of velocity, $u_t(r)$, was typical for unpermeable shells. For $C_Q = v_r(r_2)/U_0 > 10^{-4}$, in a neighborhood of the inner perforated cylinder, in a water layer 40-50 mm thick, the tangential component of the velocity was zero. The entire variation of the tangential component of the velocity from 0 to U_0 is concentrated only in a 50-60 mm thick neighborhood of the outer cylinder. This kind of flow can be regarded as a uniform turbulent boundary layer on a closed permeable surface when the pumping rate is constant. The existence of a turbulizer 1 mm in diameter on the permeable surface and held along a generator of the cylinder does not change the velocity profile.

Another characteristic feature is that the linear dimensions of this boundary layer under the conditions of the experiment depend only slightly on the rotation rate of the outer cylinder but depend to a substantial extent on the pumping coefficient. For U_0 = 22 m/sec and $C_Q = v_r(r_2)/U_0 = 2.5\cdot 10^{-4}$, we found a Reynolds number value of $Re^* = 2\cdot 10^5$ on the basis of the loss of momentum, i.e., a value 100 times as large as for the thickness of the loss of momentum of a laminar asymptotic boundary layer on a rotating permeable circular cylinder for the same value of C_Q. Measurements of the velocity profile at different cross sections along a generator showed that plane-parallel flow had taken place in the experiment.

Turbulent Flow of Low-Concentration Polymer Solutions. The phenomenon of variation in the parameters of the turbulent flow as a result of the introduction of a small amount of polymer additive is of great interest for two reasons. In the first place, an estimate of this phenomenon brings us closer to an understanding of the process of generation and dissipation of turbulence, and in the second place, it makes it possible to use this in technology. Taking account of the special features of the phenomenon, we must determine the connection between the physicochemical properties of the dissolved polymer macromolecules and the variation of the flow characteristics. The properties of the polymers which are of interest are the molecular mass, the conformation in the solution, the flexibility of the molecular chain, the degree to which it is branched, and the viscoelasticity of the macromolecular coils and their associates. Today it is impossible to change these properties by directed synthesis, and a search is in progress for new hydrodynamically effective substances of synthetic and natural origin.

An analysis of the properties of effective high polymers and a consideration of the process of evolution of fast-swimming fish have led to the assumption that mucous substances in the skin of fish belong to this class of substances [40]. It was found that there is a reduction in friction at the wall in solutions of mucus from fast-swimming and slow-swimming fish, including such swift fish as tunny (with velocities found to be as high as 20 m/sec) [41]. It was found that the effectiveness of the mucuses depends on the presence of long protein chains (which are more abundant in swift fish). The mucuses belong to the class of glycoproteins, which have as a feature of their conformation a long (asymmetric) high-polymer protein chain to which short hydrocarbon chains are attached.

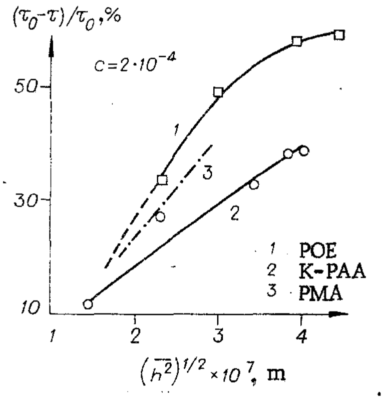

Fig. 3

Another class of biopolymers which have an asymmetric elongated conformation is that of proteoglycans. They contain long polysaccharide chains connected to short protein chains. Investigations of low-concentration solutions of natural proteoglycans secreted by various organs have shown that they have a high hydrodynamic effectiveness. Using fractionation of these substances, it was found that the effectiveness becomes higher when the molecular mass of the polysaccharide chains is greater [41]. A comparison of the chemical composition of glycoproteins and proteoglycans, as well as of the features of their conformation, leads to the conclusion that the primary (chemical) structure of the macromolecules does not influence their effectiveness to any great extent. The decisive characteristics are the great length of the polymer chains, i.e., the larger molecular mass, and their elongated (asymmetric) conformation.

The importance of the conformation of the chain was shown by the example of two different polymer solutions at constant concentration of $c = 2\cdot10^{-4}$. In experiments with a potassium salt of polyacrylic acid (K-PAA) the changes in the dimensions of the macromolecular coils were caused by different values of the pH of the solution and were directly fixed on a photogoniodiffusometer (by the method of light scattering) [42]. In experiments with polymethacrylic acid (PMA) the value of the pH was also varied, and the dimensions of the macromolecular coils were determined by the method of electron paramagnetic resonance to within a constant [43]. As can be seen from curves 2 and 3 of Fig. 3, which illustrate the reduction in friction at the wall, $(\tau_0 - \tau)/\tau_0$, the effect produced by the polymer additives on the turbulent flow increases almost linearly with increasing mean-square linear dimension of the macromolecular coils, $(\overline{h^2})^{1/2}$. It is important that both curve 2 and curve 3 were obtained from the same polymer, i.e., without changing the molecular mass. If the length of the polymer chain is increased by increasing the molecular mass, the hydrodynamic effectiveness also increases (curve 1 represents a polyoxyethylene solution).

Another problem in the mechanism of action of polymer additives on turbulence is to determine the values of polymer concentration required to achieve a specific hydrodynamic effect. From this point of view, it is of interest to make measurements of the turbulence characteristics over the entire wall layer behind the point of input of a semibounded jet of polymer solution. Measurements of the average-velocity profile, frictional stresses at the wall, and longitudinal and transverse pulsations in velocity in the large have shown that, as in the case of a uniform concentration, there is a decrease in the friction at the wall, an increase in the thickness of the viscous sublayer and the transition zone, and an increased anisotropy in the velocity pulsations in the wall region (the longitudinal pulsations change little, while the transverse pulsations become substantiallly smaller) [44].

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